

Find the general solution of the differential equation $x^2 y'' - 3xy' + 4y = \frac{x^2}{\ln x}$.

SCORE: ____ / 10 PTS

$$r^2 - 4r + 4 = 0 \rightarrow r = 2, 2 \rightarrow \underline{y_h = C_1 x^2 + C_2 x^2 \ln x}$$

$$\begin{vmatrix} x^2 & x^2 \ln x \\ 2x & 2x \ln x + x \end{vmatrix} = \underline{x^3}$$

$$\textcircled{\frac{1}{2}}$$

EACH

EXCEPT AS NOTED

$$y'' - \frac{3}{x} y' + \frac{4}{x^2} y = \frac{1}{\ln x}$$

$$y_p = -x^2 \int \frac{\left[\frac{1}{\ln x} \cdot x^2 \ln x \right]}{x^3} dx + x^2 \ln x \int \frac{\left[\frac{1}{\ln x} \cdot x^2 \right]}{x^3} dx$$

$$= -x^2 \int \frac{1}{x} dx + x^2 \ln x \int \frac{1}{x \ln x} dx$$

$\uparrow u = \ln x$

$$= \underline{-x^2 \ln |x|} + \underline{x^2 \ln x \ln |\ln x|}$$

$$y = \underline{C_1 x^2 + C_2 x^2 \ln x + x^2 \ln x \ln |\ln x|}, \textcircled{1}$$

Find the general solution of the differential equation $2y''' - 7y' + 2y = 85\sin x - 68xe^{-2x}$.

SCORE: ____ / 20 PTS

$$2r^2 - 7r + 2 = 0$$

$$\begin{array}{r|rrrr} -2 & 2 & 0 & -7 & 2 \\ & -4 & 8 & -2 & \\ \hline & 2 & -4 & 1 & 0 \end{array}$$

$$2r^2 - 4r + 1 = 0 \rightarrow r = \frac{4 \pm \sqrt{16-8}}{4} = \frac{2 \pm \sqrt{2}}{2}$$

$$y_h = c_1 e^{-2x} + c_2 e^{\frac{2+\sqrt{2}}{2}x} + c_3 e^{\frac{2-\sqrt{2}}{2}x}$$

① EACH
EXCEPT
AS NOTED

$$y_p = A \sin x + B \cos x + x(Cx + D)e^{-2x}$$

$$= A \sin x + B \cos x + (Cx^2 + Dx)e^{-2x}$$

$$y_p' = -B \sin x + A \cos x + (-2Cx^2 - 2Dx + 2Cx + D)e^{-2x}$$

$$= -B \sin x + A \cos x + (-2Cx^2 + (2C - 2D)x + D)e^{-2x} \quad \left(\frac{1}{2}\right)$$

$$y_p'' = -A \sin x - B \cos x + (4Cx^2 + (-4C + 4D)x - 2D - 4Cx + (2C - 2D))e^{-2x}$$

$$= -A \sin x - B \cos x + (4Cx^2 + (-8C + 4D)x + (2C - 4D))e^{-2x} \quad \left(\frac{1}{2}\right)$$

$$y_p''' = B \sin x - A \cos x + (-8Cx^2 + (16C - 8D)x + (-4C + 8D) + 8Cx + (-8C + 4D))e^{-2x}$$

$$= B \sin x - A \cos x + (-8Cx^2 + (24C - 8D)x + (-12C + 12D))e^{-2x} \quad \left(\frac{1}{2}\right)$$

$$2y_p''' - 7y_p' + 2y_p$$

$$= 2B \sin x - 2A \cos x + (-16Cx^2 + (48C - 16D)x + (-24C + 24D))e^{-2x}$$

$$+ 7B \sin x - 7A \cos x + (14Cx^2 + (-14C + 14D)x - 7D)e^{-2x}$$

$$+ 2A \sin x + 2B \cos x + (2Cx^2 + 2Dx)e^{-2x} \quad \left(\frac{1}{2}\right)$$

$$= (2A + 9B) \sin x + (-9A + 2B) \cos x + (34Cx + (-24C + 17D))e^{-2x}$$

$$2A + 9B = 85$$

$$-9A + 2B = 0$$

$$B = \frac{9}{2}A$$

$$2A + \frac{81}{2}A = 85$$

$$4A + 81A = 170$$

$$85A = 170 \rightarrow A = 2, B = 9$$

$$34C = -68$$

$$C = -2$$

$$48 + 17D = 0$$

$$D = -\frac{48}{17}$$

$$y = c_1 e^{-2x} + c_2 e^{\frac{2+\sqrt{2}}{2}x} + c_3 e^{\frac{2-\sqrt{2}}{2}x}$$

$$+ 2 \sin x + 9 \cos x$$

$$- (2x^2 + \frac{48}{17}x)e^{-2x} \quad \textcircled{2}$$